10.10 Take unconditional expectation on both sides of (10.41):

$$\mathrm{E}_0\left[X_t\right]=\frac{\alpha}{\beta}+\left(X_0-\frac{\alpha}{\beta}\right)\mathrm{e}^{-\beta t}+\sigma\mathrm{e}^{-\beta t}\mathrm{E}_0\left[\int_0^t \mathrm{e}^{\beta s}\mathrm{d}B_s\right].$$

By the law of iterated expectations we can write,

$$\mathrm{E}_0\left[\int_0^t \mathrm{e}^{\beta s}\mathrm{d}B_s\right]=\mathrm{E}_0\left[\int_0^t \mathrm{e}^{\beta s}\mathrm{E}_s\left[\mathrm{d}B_s\right]\right]=0,$$

and consequently we obtain

$$\mathrm{E}_0\left[X_t\right]=\frac{\alpha}{\beta}+\left(X_0-\frac{\alpha}{\beta}\right)\mathrm{e}^{-\beta t}.$$

This reveals that the long-run mean of the process is $\frac{\alpha}{\beta}$ and the rate of mean reversion is β (assuming $\beta > 0$). Figure 1 shows the expected value of the process as a function of the time distance for $\alpha=\beta=1$ and $X_0=1\pm 1$.

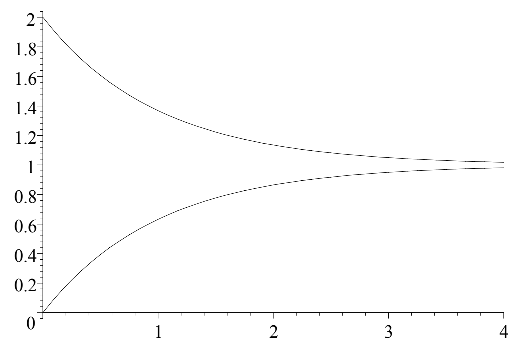

Figure 1: $\mathrm{E}_0\left[X_t\right]$ as a function of t. Starting values of X_0 are $\mu+1$ and $\mu-1$ where $\mu=1$ is the long run mean.