10.3 The essential fact is that B_s is known at time s and that the increment $B_t - B_s$ is distributed normally as of time s:

$$B_t - B_s | \mathcal{F}_s \sim N(0, t-s).$$

(a) not a martingale

$$\begin{aligned}
\mathrm{E}_s[tB_t] &= \mathrm{E}_s[t(B_s + B_t - B_s)] = \mathrm{E}_s\Big[\underbrace{tB_s}_{\text{known at } s}\Big] + \mathrm{E}_s[t(B_t - B_s)] \\
&= tB_s + t\mathrm{E}_s\Big[\underbrace{B_t - B_s}_{\text{zero conditional mean}}\Big] = tB_s \underbrace{\neq sB_s}_{\text{not a martingale}},
\end{aligned}$$

(b) martingale

$$\mathrm{E}_s[-B_t] = -\mathrm{E}_s[B_t] = -B_s,$$

(c) martingale

$$\begin{aligned}
\mathrm{E}_s\left[B_t^2 - t\right] &= \mathrm{E}_s\left[B_t^2\right] - t \\
&= \underbrace{\left(\mathrm{E}_s[B_t]\right)^2 + \mathrm{Var}_s(B_t)}_{\text{see definition of cond. variance}} - t \\
&= B_s^2 + t - s - t = B_s^2 - s
\end{aligned}$$

(d) not a martingale

$$\begin{aligned}
\mathrm{E}_s\left[B_t^3\right] &= \mathrm{E}_s\left[(B_s + B_t - B_s)^3\right] \\
&= B_s^3 + 3B_s^2 \underbrace{\mathrm{E}_s[B_t - B_s]}_{0} + 3B_s \underbrace{\mathrm{E}_s\left[(B_t - B_s)^2\right]}_{t-s} + \underbrace{\mathrm{E}_t\left[(B_t - B_s)^3\right]}_{0} \\
&= B_s^3 + 3B_s(t-s) \\
\mathrm{E}_s\left[B_t^3 - tB_t\right] &= B_s^3 + 3B_s(t-s) - tB_s = B_s^3 + 2tB_s - 3sB_s
\end{aligned}$$