

10.4 (a) Integrate both sides from s to t

$$\begin{aligned} X_t &= X_s + \int_s^t 3du + \int_s^t 7dB_u \\ &= X_s + 3(t-s) + 7 \underbrace{(B_t - B_s)}_{N(0, t-s)} \\ X_t | \mathcal{F}_s &\sim N(X_s + 3(t-s), 49(t-s)). \end{aligned}$$

The process X is Markov.

(b)

$$\begin{aligned} X_t &= X_s + \underbrace{\int_s^t 2udu}_{t^2 - s^2} - \underbrace{\int_s^t u dB_u}_{N(0, \int_s^t u^2 du)} \\ X_t | \mathcal{F}_s &\sim N\left(X_s + t^2 - s^2, \frac{t^3 - s^3}{3}\right) \end{aligned}$$

The process X is Markov.

(c) First use the Ito formula to find the SDE for $\ln X$ and then integrate from s to t

$$\begin{aligned} d \ln X_t &= \left(0.15 - \frac{1}{2}0.2^2\right) dt + 0.2dB_t \\ \ln X_t &= \ln X_s + \underbrace{\int_s^t 0.13du}_{0.13(t-s)} + \underbrace{\int_s^t 0.2dB_u}_{N(0, 0.04(t-s))} \\ \ln X_t | \mathcal{F}_s &\sim N(\ln X_s + 0.13(t-s), 0.04(t-s)). \end{aligned}$$

This means X_t is distributed lognormally, the mean and variance of the lognormal distribution can be found on page 361 (Section B.12) of the MTF book:

$$X_t | \mathcal{F}_s \sim \log N(X_s e^{0.15(t-s)}, X_s^2 [e^{0.34(t-s)} - e^{0.3(t-s)}]).$$

The process X is Markov.