10.7 By the Itô formula

$$\mathrm{d}(\mathrm{e}^{\beta t}X_t) = \beta \mathrm{e}^{\beta t}X_t \mathrm{d}t + \mathrm{e}^{\beta t}\mathrm{d}X_t.$$

Substitute for $\mathrm{d}X_t$ and simplify

$$\mathrm{d}(\mathrm{e}^{\beta t}X_t) = \left(\beta \mathrm{e}^{\beta t}X_t + \alpha \mathrm{e}^{\beta t} - \beta \mathrm{e}^{\beta t}X_t\right)\mathrm{d}t + \sigma \mathrm{e}^{\beta t}\mathrm{d}B_t$$
$$\mathrm{d}(\mathrm{e}^{\beta t}X_t) = \alpha \mathrm{e}^{\beta t}\mathrm{d}t + \sigma \mathrm{e}^{\beta t}\mathrm{d}B_t. \tag{1}$$

Consequently the process

$$Z_t = \mathrm{e}^{\beta t}X_t \tag{2}$$

is a Brownian motion with deterministic drift and volatility. Integrate both sides of (1) from 0 to T,

$$\mathrm{e}^{\beta T}X_T - \mathrm{e}^{0}X_0 = \int_0^T \alpha \mathrm{e}^{\beta t}\mathrm{d}t + \int_0^T \sigma \mathrm{e}^{\beta t}\mathrm{d}B_t,$$

and solve for X_T

$$\begin{aligned} X_T &= \mathrm{e}^{-\beta T}X_0 + \int_0^T \alpha \mathrm{e}^{\beta(t-T)}\mathrm{d}t + \int_0^T \sigma \mathrm{e}^{\beta(t-T)}\mathrm{d}B_t \\ &= \frac{\alpha}{\beta} + \mathrm{e}^{-\beta T}\left(X_0 - \frac{\alpha}{\beta}\right) + \int_0^T \sigma \mathrm{e}^{\beta(t-T)}\mathrm{d}B_t. \end{aligned}$$

Note that this is precisely the formula (10.41) in the text.