10.9 From the formula (10.41)

$$X_t = \frac{\alpha}{\beta} + \left(X_0 - \frac{\alpha}{\beta}\right) \mathrm{e}^{-\beta t} + \sigma \mathrm{e}^{-\beta t} \int_0^t \mathrm{e}^{\beta s} \mathrm{d}B_s.$$

With $t = 1$ we deduce that

$$\begin{aligned} X_1 | \mathcal{F}_0 &\sim N\left(\frac{\alpha}{\beta} + \left(X_0 - \frac{\alpha}{\beta}\right) \mathrm{e}^{-\beta}, \sigma^2 \mathrm{e}^{-2\beta} \int_0^1 \mathrm{e}^{2\beta u} \mathrm{d}u\right) \\ &\sim N\left(\frac{\alpha}{\beta} + \left(X_0 - \frac{\alpha}{\beta}\right) \mathrm{e}^{-\beta}, \sigma^2 \frac{1 - \mathrm{e}^{-2\beta}}{2\beta}\right) \end{aligned}$$

We can rephrase this as follows

$$X_1 = \frac{\alpha}{\beta}\left(1 - \mathrm{e}^{-\beta}\right) + X_0 \mathrm{e}^{-\beta} + \sigma \sqrt{\frac{1 - \mathrm{e}^{-2\beta}}{2\beta}} \varepsilon_1,$$

where ε_1 is a standard normal variable. Comparison of the above with the discrete time version $X_1 = \mu + \rho X_0 + \tilde{\sigma} \varepsilon_1$ yields:

$$\begin{aligned} \mu &= \frac{\alpha}{\beta}\left(1 - \mathrm{e}^{-\beta}\right), \\ \rho &= \mathrm{e}^{-\beta}, \\ \tilde{\sigma} &= \sigma \sqrt{\frac{1 - \mathrm{e}^{-2\beta}}{2\beta}}. \end{aligned}$$

For β small we can use the first order Taylor expansion to find an approximate relationship:

$$\begin{aligned} \mu &\approx \alpha\left(1 - \frac{\beta}{2}\right), \\ \rho &\approx 1 - \beta, \\ \tilde{\sigma} &\approx \sigma\left(1 - \frac{\beta}{2}\right). \end{aligned}$$