

**8.3** To calculate the conditional expectations, one needs to know the conditional probabilities of moving in the decision tree. These can be worked out backwards, from the unconditional probabilities of passing through a specific node, see Figure 1.

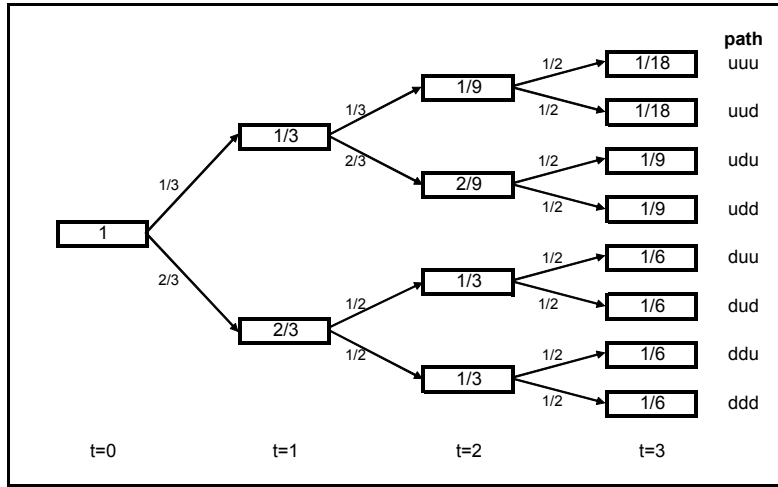


Figure 1: One-step conditional probabilities are at the branches; the number at each node signifies the unconditional probability of passing through that node.

With the one-step conditional probabilities in hand we can work backwards to obtain  $E_2[X]$ , then  $E_1[X] = E_1[E_2[X]]$  and finally  $E_0[X] = E_0[E_1[X]]$ . Since  $E_1[X]$  is known at  $t = 1$  it is also known at  $t = 2$ , therefore  $E_2[E_1[X]] = E_1[X]$ . For numerical values see Table 1.

outcome	$P(\{\omega\})$	$X$	$E_2[X]$	$E_1[X]$	$E_2[E_1[X]]$	$E_1[E_2[X]]$	$E_0[X]$
uuu	$\frac{1}{18}$	8	7.5	$6\frac{1}{6}$	$6\frac{1}{6}$	$6\frac{1}{6}$	$3\frac{13}{18}$
uud	$\frac{1}{18}$	7	7.5	$6\frac{1}{6}$	$6\frac{1}{6}$	$6\frac{1}{6}$	$3\frac{13}{18}$
udu	$\frac{1}{9}$	6	5.5	$6\frac{1}{6}$	$6\frac{1}{6}$	$6\frac{1}{6}$	$3\frac{13}{18}$
udd	$\frac{1}{9}$	5	5.5	$6\frac{1}{6}$	$6\frac{1}{6}$	$6\frac{1}{6}$	$3\frac{13}{18}$
duu	$\frac{1}{6}$	4	3.5	2.5	2.5	2.5	$3\frac{13}{18}$
dud	$\frac{1}{6}$	3	3.5	2.5	2.5	2.5	$3\frac{13}{18}$
ddu	$\frac{1}{6}$	2	1.5	2.5	2.5	2.5	$3\frac{13}{18}$
ddd	$\frac{1}{6}$	1	1.5	2.5	2.5	2.5	$3\frac{13}{18}$

Table 1: Conditional expectations