B.16 Let us denote by Σ^Y the covariance matrix of Y_1, Y_2 and Y_3. Recall that the correlation is defined as

$$\mathrm{Corr}(Y_i, Y_j) = \frac{\mathrm{Cor}(Y_i, Y_j)}{\sqrt{\mathrm{Var}(Y_i)}\sqrt{\mathrm{Var}(Y_j)}} = \frac{\Sigma^Y_{ij}}{\sqrt{\Sigma^Y_{ii}}\sqrt{\Sigma^Y_{jj}}}.$$

In matrix form this can be conveniently written as

$$\mathrm{Corr}^Y = \begin{bmatrix} \frac{1}{\sqrt{\Sigma^Y_{11}}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{\Sigma^Y_{22}}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{\Sigma^Y_{33}}} \end{bmatrix} \Sigma^Y \begin{bmatrix} \frac{1}{\sqrt{\Sigma^Y_{11}}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{\Sigma^Y_{22}}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{\Sigma^Y_{33}}} \end{bmatrix}.$$

All we need to do is to find out the covariance matrix Σ^Y.

Variables Y_1, Y_2 and Y_3 are linear combinations of X_1 and X_2, namely

$$\begin{aligned} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} &= \begin{bmatrix} 0.25 & 0.75 \\ 0.5 & 0.5 \\ \alpha & 1-\alpha \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \\ Y &= AX. \end{aligned}$$

By virtue of the portfolio theorem we have therefore,

$$\begin{aligned} \Sigma^Y &= A\Sigma^X A^* \\ &= \begin{bmatrix} 0.25 & 0.75 \\ 0.5 & 0.5 \\ \alpha & 1-\alpha \end{bmatrix} \begin{bmatrix} 0.01 & -0.01 \\ -0.01 & 0.04 \end{bmatrix} \begin{bmatrix} 0.25 & 0.5 & \alpha \\ 0.75 & 0.5 & 1-\alpha \end{bmatrix} \\ &= \begin{bmatrix} 0.019375 & 0.01125 & 0.0275 - 0.0325\alpha \\ 0.01125 & 0.0075 & 0.015(1-\alpha) \\ 0.0275 - 0.0325\alpha & 0.015(1-\alpha) & 0.04 - 0.1\alpha + 0.07\alpha^2 \end{bmatrix} \qquad (1) \end{aligned}$$

(a) From the preceding result (1) we can deduce that

$$\mathrm{Corr}(Y_1, Y_2) = \frac{\mathrm{Cov}(Y_1, Y_2)}{\sqrt{\mathrm{Var}(Y_1)}\sqrt{\mathrm{Var}(Y_2)}} = \frac{0.01125}{\sqrt{0.019375}\sqrt{0.0075}} = 0.933.$$

(b) Y_2 and Y_3 will be uncorrelated if and only if $\mathrm{Cov}(Y_2, Y_3) = 0$. We know from (1) that

$$\mathrm{Cov}(Y_2, Y_3) = 0.015(1-\alpha),$$

and this can only be equal to zero if $\alpha = 1$.

(c) When $\alpha = 0.75$ we obtain

$$\Sigma^Y = \begin{bmatrix} 0.019375 & 0.01125 & 0.003125 \\ 0.01125 & 0.0075 & 0.00375 \\ 0.003125 & 0.00375 & 0.004375 \end{bmatrix},$$

and

$$\mathrm{Corr}^Y = \begin{bmatrix} 1 & 0.933 & 0.339 \\ 0.933 & 1 & 0.655 \\ 0.339 & 0.655 & 1 \end{bmatrix}.$$

(d) In GAUSS define:

```
alpha=0.75
A=(0.25~0.75)|(0.5~0.5)|(alpha~1-alpha);
SigX=(0.01~-0.01)|(-0.01~0.04);
SigY=A*SigX*A';
stdY=sqrt(diag(SigY)) /* vector of st deviations */;
corrY=SigY./stdY./stdY';
```