B.20 (a) The marginal cumulative distributions of X and Y are obtained by letting y and x, respectively, to go to infinity:

$$\begin{aligned}
F_X(x) &= \lim_{y\to\infty} \frac{\exp(2x+3y)}{(1+\exp(x))^2\,(1+\exp(3y))} \\
&= \frac{\exp(2x)}{(1+\exp(x))^2} \lim_{y\to\infty} \frac{\exp(3y)}{1+\exp(3y)} \\
&= \frac{\exp(2x)}{(1+\exp(x))^2} \\
F_Y(y) &= \lim_{x\to\infty} \frac{\exp(2x+3y)}{(1+\exp(x))^2\,(1+\exp(3y))} \\
&= \frac{\exp(3y)}{1+\exp(3y)} \lim_{x\to\infty} \frac{\exp(2x)}{(1+\exp(x))^2} \\
&= \frac{\exp(3y)}{1+\exp(3y)}
\end{aligned}$$

Since $F_{X,Y}(x,y) = F_X(x) \times F_Y(y)$ we conclude that X and Y are independent.

(b) The joint density is obtained as the second partial derivative of the cumulative distribution function:

$$\begin{aligned}
f_{X,Y}(x,y) &= \frac{\partial^2 F_{X,Y}(x,y)}{\partial x \partial y} \\
&= \frac{\partial}{\partial x}\left(\frac{\exp(2x)}{(1+\exp(x))^2}\frac{3\exp(3y)}{(1+\exp(3y))^2}\right) \\
&= \frac{2\exp(2x)}{(1+\exp(x))^3}\frac{3\exp(3y)}{(1+\exp(3y))^2}.
\end{aligned}$$