B.23 To exploit the normality of log stock price we will rewrite S_T^γ as an exponential:

$$\mathrm{E}^Q\left[S_T^\gamma\right] = \mathrm{E}^Q\left[\mathrm{e}^{\gamma \ln S_T}\right].$$

If $\ln S_T \overset{Q}{\sim} N(m, s^2)$, then from the moment generating function of normal distribution we have

$$\mathrm{E}^Q\left[e^{\gamma \ln S_T}\right] = \exp\left(\gamma m + \frac{1}{2}\gamma^2 s^2\right).$$

Substituting $m = \ln S_0 + \left(r - \frac{\sigma^2}{2}\right) T$ and $s^2 = \sigma^2 T$ we obtain the no-arbitrage price of the power contract:

$$\begin{aligned} e^{-rT}\mathrm{E}^Q\left[S_T^\gamma\right] &= S_0^\gamma \exp\left((\gamma - 1)rT + \frac{\sigma^2}{2}T\left(\gamma^2 - \gamma\right)\right) \\ &= S_0^\gamma \exp\left((\gamma - 1)T\left(r + \frac{\gamma\sigma^2}{2}\right)\right). \end{aligned}$$