

B.25 Let $x_1 < x_2$ be two neighbouring values of X and $y_1 < y_2$ be two neighbouring values of Y . Then

$$P(X = x_2, Y = y_2) = F(x_2, y_2) - F(x_2, y_1) - F(x_1, y_2) + F(x_1, y_1),$$

see Section B.9.3. Employing the independence of X and Y we obtain

$$\begin{aligned} P(X = x_2, Y = y_2) &= F_X(x_2)F_Y(y_2) - F_X(x_2)F_Y(y_1) - F_X(x_1)F_Y(y_2) + F_X(x_1)F_Y(y_1) \\ &= F_X(x_2)(F_Y(y_2) - F_Y(y_1)) - F_X(x_1)(F_Y(y_2) - F_Y(y_1)) \\ &= (F_X(x_2) - F_X(x_1))(F_Y(y_2) - F_Y(y_1)) \\ &= P(X = x_2)P(Y = y_2). \end{aligned}$$