Contents

Preface to the Second Edition			xiii	
From the Preface to the First Edition				
1	The S	e Simplest Model of Financial Markets		
	1.1	One-Period Finite State Model	1	
	1.2	Securities and Their Payoffs	3	
	1.3	Securities as Vectors	3	
	1.4	Operations on Securities	4	
	1.5	The Matrix as a Collection of Securities	6	
	1.6	Transposition	6	
	1.7	Matrix Multiplication and Portfolios	8	
	1.8	Systems of Equations and Hedging	10	
	1.9	Linear Independence and Redundant Securities	12	
	1.10	The Structure of the Marketed Subspace	14	
	1.11	The Identity Matrix and Arrow–Debreu Securities	16	
	1.12	Matrix Inverse	17	
	1.13	Inverse Matrix and Replicating Portfolios	17	
	1.14	Complete Market Hedging Formula	19	
	1.15	Summary	20	
	1.16	Notes	21	
	1.17	Exercises	22	
2	2 Arbitrage and Pricing in the One-Period Model		25	
	2.1	Hedging with Redundant Securities and Incomplete Market	25	
	2.2	Finding the Best Approximate Hedge	29	
	2.3	Minimizing the Expected Squared Replication Error	32	
	2.4	Numerical Stability of Least Squares	34	
	2.5	Asset Prices, Returns and Portfolio Units	36	
	2.6	Arbitrage	38	
	2.7	No-Arbitrage Pricing	40	
	2.8	State Prices and the Arbitrage Theorem	41	
	2.9	State Prices and Asset Returns	44	
	2.10	Risk-Neutral Probabilities	45	
	2.11	State Prices and No-Arbitrage Pricing	46	
	2.12	Asset Pricing Duality	47	
	2.13	Summary	48	
	2.14	Notes	49	
	2.15	Appendix: Least Squares with QR Decomposition	49	
	2.16	Exercises	52	

x		Co	ntents			
3	Risk and Return in the One-Period Model 55					
	3.1	Utility Functions	56			
	3.2	Expected Utility Maximization	59			
	3.3	The Existence of Optimal Portfolios	61			
	3.4	Reporting Expected Utility in Terms of Money	62			
	3.5	Normalized Utility and Investment Potential	63			
	3.6	Quadratic Utility	67			
	3.7	The Sharpe Ratio	69			
	3.8	Arbitrage-Adjusted Sharpe Ratio	71			
	3.9	The Importance of Arbitrage Adjustment	75			
	3.10	Portfolio Choice with Near-Arbitrage Opportunities	//			
	3.11	Summary	/9			
	3.12 2.12	Notes Exercises	81			
	5.15	Exercises	62			
4	Num	erical Techniques for Optimal Portfolio Selection in Incomplete Market	s 84			
	4.1	Sensitivity Analysis of Portfolio Decisions with the CRRA Utility	84			
	4.2	Newton's Algorithm for Optimal Investment with CRRA Utility	88			
	4.3	Optimal CRRA Investment Using Empirical Return Distribution	90			
	4.4	HARA Portfolio Optimizer	94			
	4.5	HARA PORTIONO Optimization with Several Risky Assets	90			
	4.0	Quadratic Outity Maximization with Multiple Assets	102			
	4.7	Notes	102			
	4.0 4.9	Fxercises	102			
	1.7		102			
5	Prici	ng in Dynamically Complete Markets	104			
	5.1	Options and Portfolio Insurance	104			
	5.2	Option Pricing	105			
	5.3	Dynamic Replicating Trading Strategy	108			
	5.4	Risk-Neutral Probabilities in a Multi-Period Model	110			
	5.5	The Law of Iterated Expectations	119			
	5.0	Summary	121			
	5.1	Notes Exercises	121			
	5.8	Exercises	121			
6	Towa	rds Continuous Time	125			
	6.1	IID Returns, and the Term Structure of Volatility	125			
	6.2	Towards Brownian Motion	127			
	6.3	Towards a Poisson Jump Process	136			
	6.4	Central Limit Theorem and Infinitely Divisible Distributions	142			
	0.3	Summary	145			
	6.7	Exercises	145			
-	De 14	Earston Thomas forme	1.47			
1	rast	rourier transform	147			
	7.1 7.2	Discrete Fourier Transform (DET)	14/			
	1.2 7 2	Fourier Transforms in Finance	152			
	7.3 7.4	Fast Pricing via the East Fourier Transform (FET)	155			
	7. 4 7.5	Further Applications of FFTs in Finance	162			
	7.5 7.6	Notes	166			
	7.7	Appendix	167			
	7.8	Exercises	169			

<i>Contents</i> x				
8	Inform	mation Management	170	
	8.1	Information: Too Much of a Good Thing?	170	
	8.2	Model-Independent Properties of Conditional Expectation	174	
	8.3	Summary	178	
	8.4	Notes	179	
	8.5	Appendix: Probability Space	179	
	8.6	Exercises	183	
0	Marti	ingeles and Change of Measure in Finance	187	
,	91	Discounted Asset Prices Are Martingales	187	
	9.1	Dynamic Arbitrage Theorem	107	
	9.2	Change of Measure	103	
	94	Dynamic Ontimal Portfolio Selection in a Complete Market	198	
	9.5	Summary	206	
	9.6	Notes	208	
	9.7	Exercises	208	
10	D	and the second telephone in the	010	
10	Brow	nian Iviouon and ito Formulae	213	
	10.1	Continuous- 11me Brownian Motion	213	
	10.2	Stochastic Integration and Ito Processes	218	
	10.3	Important Ito Processes	220	
	10.4	Function of a Stochastic Process: the Ito Formula	222	
	10.5	Applications of the Ito Formula	223	
	10.6		223	
	10.7	Ito Processes as Martingales	228	
	10.8		229	
	10.9	Summary	229	
	10.10	Notes	230	
	10.11	Exercises	251	
11	Conti	nuous-Time Finance	233	
	11.1	Summary of Useful Results	233	
	11.2	Risk-Neutral Pricing	234	
	11.3	The Girsanov Theorem	237	
	11.4	Risk-Neutral Pricing and Absence of Arbitrage	241	
	11.5	Automatic Generation of PDEs and the Feynman–Kac Formula	246	
	11.6	Overview of Numerical Methods	250	
	11.7	Summary	251	
	11.8	Notes	252	
	11.9	Appendix: Decomposition of Asset Returns into Uncorrelated Components	252	
	11.10	Exercises	255	
12	Finite	e-Difference Methods	261	
	12.1	Interpretation of PDEs	261	
	12.2	The Explicit Method	263	
	12.3	Instability	264	
	12.4	Markov Chains and Local Consistency	266	
	12.5	Improving Convergence by Richardson's Extrapolation	268	
	12.6	Oscillatory Convergence Due to Grid Positioning	269	
	12.7	Fully Implicit Scheme	270	
	12.8	Crank–Nicolson Scheme	273	
	12.9	Summary	274	
	12.10	Notes	276	
	12.11	Appendix: Efficient Gaussian Elimination for Tridiagonal Matrices	276	

xii		Сон	itents			
	12.12 12.13	Appendix: Richardson's Extrapolation Exercises	277 277			
13	Dvnai	nic Option Hedging and Pricing in Incomplete Markets	280			
	13.1	The Risk in Option Hedging Strategies	280			
	13.2	Incomplete Market Option Price Bounds	299			
	13.3	Towards Continuous Time	304			
	13.4	Derivation of Optimal Hedging Strategy	309			
	13.5	Summary	318			
	13.6	Notes	319			
	13.7	Appendix: Expected Squared Hedging Error in the Black–Scholes Model	320			
	13.8	Exercises	322			
Ap	pendix	A Calculus	326			
r	A.1	Notation	326			
	A.2	Differentiation	329			
	A.3	Real Function of Several Real Variables	332			
	A.4	Power Series Approximations	334			
	A.5	Optimization	336			
	A.6	Integration	338			
	A.7	Exercises	344			
Ap	pendix	B Probability	348			
	B.1	Probability Space	348			
	B.2	Conditional Probability	348			
	B.3	Marginal and Joint Distribution	351			
	B.4	Stochastic Independence	352			
	B.5	Expectation Operator	354			
	B.6	Properties of Expectation	355			
	B.7	Mean and Variance	356			
	B.8	Covariance and Correlation	357			
	B.9	Continuous Random Variables	360			
	B.10	Normal Distribution	364			
	B.11	Quantiles	370			
	B.12	Relationships among Standard Statistical Distributions	371			
	B.13	Notes	372			
	B.14	Exercises	372			
Re	References					
Inc	Index					